The Arrow of Time and the Page Curve

Jean-Paul Niko

2026

The Arrow of Time and the Page Curve

Derived from GL Condensate Dynamics

Jean-Paul Niko – 2026 – RTSG-2026-010 Mathematical supplement to RTSG-2026-009.

Abstract

Two calculations derived from the GL condensate framework. (1) The arrow of time as monotonically increasing entanglement entropy during Big Bang condensation, supported by the Komargodski-Schwimmer a-theorem and 1+1D quantum quench results. (2) The Page curve from the island formula applied to the GL condensate phase boundary, giving t_Page ~ S0/kappa = t_kin. Information preservation via unitarity of the full GL partition function.

I. The Arrow of Time

I.1 GL Fixed Points

The GL theory has a conformal fixed point at T = T_c. UV fixed point: c = 1 (complex boson, massless). IR fixed point: c = 0 (gapped, massive, condensed).

The KS a-theorem (Komargodski-Schwimmer 2011):

```
dC/d(ln mu) <= 0
```

RG flow from T_c to T=0 is monotone. UV degrees of freedom freeze out.

I.2 SSA Correction

FATAL ERROR IN EARLIER VERSION – CORRECTED HERE:

Strong subadditivity states: S(AuB) + S(AnB) <= S(A) + S(B)

With S(AnB) = 0 before a domain merge, this gives: S(AuB) <= S(A) + S(B)

This is an UPPER bound. The earlier claim that domain merges are "superadditive in entanglement" was direction-reversed. SSA cannot prove dS_ent/dt >= 0. The argument is retracted.

I.3 Correct Status

dS_ent/dt >= 0 is supported by: (a) KS a-theorem: RG monotonicity (established) (b) 1+1D quantum quench calculations: exact results in lower-dimensional analogues [Calabrese-Cardy 2005] (c) Physical intuition: long-range correlations grow during condensation

Full proof for 3+1D GL condensate requires the quantum quench calculation. This is the primary open mathematical task.

II. The Page Curve

II.1 Island Formula

```
S(R,t) = min[ S_thermal(t),  A(dI)/4G + S_matter(R u I) ]
```

Note on applicability: The island formula requires a holographic setup in asymptotically AdS geometry. The GL condensate lives in asymptotically flat or de Sitter space. Direct application requires a bespoke derivation – this is an open problem (replica wormhole calculation in GL framework).

Early times: S(R,t) ~ T_H * t * ln2 [rising] Late times: S(R,t) ~ S0 - T_H$t$ln2 [falling, island dominates]

II.2 Page Time

```
t_Page = pi*S0 / (kappa * ln2) ~ S0/kappa = t_kin
```

The Page time equals the GL kinematic clock. The condensate relaxation timescale and the Page time are the same object.

II.3 Information – Corrected

CATEGORY ERROR IN EARLIER VERSION – CORRECTED HERE:

Earlier claim: "U(1) conservation = information preservation." This is wrong. U(1) conserves one Noether charge, not the Hilbert space.

Correct mechanism: 1. Unitarity of Z = int DW0 exp(-S[W0]) 2. Replica wormhole saddles in Euclidean GL path integral restore Page curve 3. U(1) winding conservation encodes topological sector information only

III. The Connection

Big Bang (Section I) and black hole evaporation (Section II) are the same GL field traversing the Mexican-hat potential in opposite directions. Not a symmetry. Not a time-reversal. A phase inversion.

The global arrow of time is preserved because Big Bang condensation continues everywhere, while black hole decondensation is local.

IV. Open Problems

1. 3+1D quantum quench calculation for dS_ent/dt >= 0
2. Island formula derivation for asymptotically flat GL background
3. Explicit replica wormhole calculation in GL Euclidean path integral

References

[1] Z. Komargodski and A. Schwimmer, JHEP 12, 099 (2011), arXiv:1107.3987. [2] A. Almheiri et al., Rev. Mod. Phys. 93, 035002 (2021), arXiv:2006.06872. [3] G. Penington et al., "Replica Wormholes," arXiv:1911.11977 (2019). [4] P. Calabrese and J. Cardy, J. Stat. Mech. P04010 (2005). [5] T. W. B. Kibble, J. Phys. A 9, 1387 (1976).