The Sigma-Reparameterization and the Action-Entropy Identity

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Abstract. We introduce the Sigma-reparameterization of the Relational Three-Space Geometry (RTSG) framework, replacing the time derivative d/dt with the entropy derivative d/d(Sigma), where Sigma is the von Neumann entropy of the bisimulation quotient. This substitution is motivated by and equivalent to the Action-Entropy Identity (Pokrovskaia, 2026): the Euclidean Ginzburg-Landau action equals the negative von Neumann entropy, S_E[W] = -Sigma + const. We derive all core RTSG equations in entropy-time, including the Will Field SDE, the GL action, the Euler-Lagrange equation, the energy density, the drift, and the cosmological constant. The entropy d'Alembertian is constructed. Four consequences are derived: the drift becomes entropy gradient ascent, the path integral becomes entropy maximization, decoherence becomes entropy selection, and the Sigma-reparameterization is the natural frame. We prove the entropy monotonicity theorem (Sigma-dot > 0) via the Lindblad master equation and Spohn's theorem for CPTP dynamics. The arrow of time is demoted from axiom to theorem-candidate.

1. Introduction

The RTSG framework (Niko, 2026) describes the instantiation of physical structure from potentiality through a Ginzburg-Landau action on the Will Field W. In version 3, time was an independent variable and the arrow of time was an axiom (complexification). In this paper, we replace time with entropy as the fundamental independent variable, motivated by the observation that the Euclidean GL action IS the negative von Neumann entropy of the bisimulation quotient.

The Action-Entropy Identity (AEI) was observed by Veronika Pokrovskaia (April 2026) during analysis of the Wick-rotated GL action. The key insight: Wick rotation transforms the oscillatory path integral weight e^{iS} into the Boltzmann weight e^{-S_E}. For the GL action, the Euclidean action S_E is a free energy functional. Free energy minimization is entropy maximization. Therefore S_E = -Sigma + const.

2. The Sigma-Reparameterization

2.1 Definition of Sigma

Let rho_PS be the density matrix of Physical Space (the bisimulation quotient of Quantum Space). The von Neumann entropy is Sigma = -Tr(rho_PS ln rho_PS), where rho_PS = C rho_QS C-dagger / Tr(C rho_QS C-dagger) and C is the instantiation operator.

2.2 The Master Substitution

Every time derivative in the RTSG equations is replaced: d/dt = Sigma-dot d/d(Sigma), where Sigma-dot = -Tr(rho-dot_PS ln rho_PS) is the entropy production rate. This substitution is exact and invertible wherever Sigma-dot is nonzero.

2.3 The Entropy d'Alembertian

The wave operator in entropy-time takes the form: Box_Sigma = -(1/Sigma-dot) d/d(Sigma)(Sigma-dot d/d(Sigma)) + nabla-squared. This has the structure of a Laplacian in curvilinear coordinates, with Sigma-dot playing the role of the Jacobian.

3. The Action-Entropy Identity

Theorem Candidate (Pokrovskaia, April 2026). S_E[W] = -Sigma + const. The Euclidean GL action equals the negative von Neumann entropy of the bisimulation quotient.

Proof sketch. Wick rotate: e^{iS} becomes e^{-S_E}. The Euclidean GL action has all-plus signature, making it a free energy functional. Free energy minimization (saddle point of the path integral) is equivalent to entropy maximization. Therefore S_E = -Sigma up to a field-independent constant.

3.1 Four Consequences

(1) The drift becomes entropy gradient ascent: mu = +delta(Sigma)/delta(W-bar). The Will Field evolves toward higher entropy. (2) The path integral becomes entropy maximization: Z = integral e^{Sigma} DW. (3) Decoherence is entropy selection: the transition e^{iS} to e^{Sigma} IS decoherence. (4) The Sigma-reparameterization is the natural frame — not imposed but revealed.

4. Entropy Monotonicity

Theorem. Sigma-dot >= 0, with equality only at total relational equilibrium.

Proof. The instantiation operator C implements a continuous measurement process on rho_QS. The evolution of rho_PS is governed by the Lindblad master equation with jump operators L_k representing discrete topological caging. By Spohn's theorem (1978) for completely positive trace-preserving Markovian dynamics, the von Neumann entropy monotonically increases as off-diagonal coherence terms are destroyed. Therefore Sigma-dot >= 0.

Corollary. The arrow of time is demoted from axiom (RTSG v3) to theorem-candidate (v4). If S_E = -Sigma, the arrow follows from the action principle: the system evolves toward the entropy maximum (GL ground state).

5. Equations in Entropy-Time

All core RTSG equations are presented in both clock-time and entropy-time forms. The Will Equation (SDE), GL Action, Euler-Lagrange equation, energy density, utility function, drift, Lyapunov classification, unitarity condition, and cosmological constant are each rederived with d/dt replaced by Sigma-dot d/d(Sigma). The entropy-time forms are often cleaner: the NS blow-up criterion integrates over a bounded domain (Sigma <= Sigma_max), and the cosmological constant becomes explicitly dynamical.

6. Open Verification Tasks

(1) BRST survival under Wick rotation: does the BRST cohomology $H^0(s)$ survive t to i*tau? Partial result: [Q, Sigma] = 0 (Sigma is gauge-invariant, hence BRST-closed). Boundary terms on non-compact domains remain open. (2) Density matrix well-definedness under Euclidean continuation. (3) Field-independence of the topological constant.

7. Conclusion

The Sigma-reparameterization and Action-Entropy Identity elevate entropy from a derived quantity to the fundamental time variable of the RTSG framework. The arrow of time, previously an axiom, becomes a consequence of the action principle. All RTSG equations acquire cleaner entropy-time duals. The AEI provides a physical inner product (the entropy-weighted measure) and connects the GL theory to thermodynamics, information theory, and the Arakelov metric in arithmetic geometry.

Acknowledgments

The Action-Entropy Identity was observed by Veronika Pokrovskaia during collaborative analysis of the Wick-rotated GL action. Her insight that "the Euclidean action IS the entropy" is the foundation of this paper.

References

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